

NAG Toolbox for MATLAB

g13dx

1 Purpose

g13dx calculates the zeros of a vector autoregressive (or moving average) operator. This function is likely to be used in conjunction with g05pc, g13as, g13dc or g13ds.

2 Syntax

```
[rr, ri, rmod, ifail] = g13dx(k, ip, par)
```

3 Description

Consider the vector autoregressive moving average (VARMA) model

$$W_t - \mu = \phi_1(W_{t-1} - \mu) + \phi_2(W_{t-2} - \mu) + \cdots + \phi_p(W_{t-p} - \mu) + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \cdots - \theta_q\epsilon_{t-q}, \quad (1)$$

where W_t denotes a vector of k time series and ϵ_t is a vector of k residual series having zero mean and a constant variance-covariance matrix. The components of ϵ_t are also assumed to be uncorrelated at non-simultaneous lags. $\phi_1, \phi_2, \dots, \phi_p$ denotes a sequence of k by k matrices of autoregressive (AR) parameters and $\theta_1, \theta_2, \dots, \theta_q$ denotes a sequence of k by k matrices of moving average (MA) parameters. μ is a vector of length k containing the series means. Let

$$A(\phi) = \begin{bmatrix} \phi_1 & I & 0 & . & . & . & 0 \\ \phi_2 & 0 & I & 0 & . & . & 0 \\ . & . & . & . & . & . & . \\ \phi_{p-1} & 0 & . & . & . & 0 & I \\ \phi_p & 0 & . & . & . & 0 & 0 \end{bmatrix}_{pk \times pk}$$

where I denotes the k by k identity matrix.

The model (1) is said to be stationary if the eigenvalues of $A(\phi)$ lie inside the unit circle. Similarly let

$$B(\theta) = \begin{bmatrix} \theta_1 & I & 0 & . & . & . & 0 \\ \theta_2 & 0 & I & 0 & . & . & 0 \\ . & . & . & . & . & . & . \\ \theta_{q-1} & 0 & . & . & . & 0 & I \\ \theta_q & 0 & . & . & . & 0 & 0 \end{bmatrix}_{qk \times qk}$$

Then the model is said to be invertible if the eigenvalues of $B(\theta)$ lie inside the unit circle.

g13dx returns the pk eigenvalues of $A(\phi)$ (or the qk eigenvalues of $B(\theta)$) along with their moduli, in descending order of magnitude. Thus to check for stationarity or invertibility you should check whether the modulus of the largest eigenvalue is less than one.

4 References

Wei W W S 1990 *Time Series Analysis: Univariate and Multivariate Methods* Addison–Wesley

5 Parameters

5.1 Compulsory Input Parameters

1: **k** – int32 scalar

k , the dimension of the multivariate time series.

Constraint: $k \geq 1$.

2: **ip** – int32 scalar

The number of AR (or MA) parameter matrices, p (or q).

Constraint: $ip \geq 1$.

3: **par**($ip \times k \times k$) – double array

The AR (or MA) parameter matrices read in row by row in the order $\phi_1, \phi_2, \dots, \phi_p$ (or $\theta_1, \theta_2, \dots, \theta_q$). That is, **par**(($l-1$) $\times k \times k + (i-1) \times k + j-1$) must be set equal to the (i,j) th element of ϕ_l , for $l = 1, 2, \dots, p$ (or the (i,j) th element of θ_l , for $l = 1, 2, \dots, q$).

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

work, iwork

5.4 Output Parameters

1: **rr**($k \times ip$) – double array

The real parts of the eigenvalues.

2: **ri**($k \times ip$) – double array

The imaginary parts of the eigenvalues.

3: **rmod**($k \times ip$) – double array

The moduli of the eigenvalues.

4: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **k** < 1,
or **ip** < 1.

ifail = 2

An excessive number of iterations are needed to evaluate the eigenvalues of $A(\phi)$ (or $B(\theta)$). This is an unlikely exit. All output parameters are undefined.

7 Accuracy

The accuracy of the results depends on the original matrix and the multiplicity of the roots.

8 Further Comments

The time taken is approximately proportional to kp^3 (or kq^3).

9 Example

```
k = int32(2);
ip = int32(1);
par = [0.802;
       0.065;
       0;
       0.575];
[rr, ri, rmod, ifail] = g13dx(k, ip, par)

rr =
    0.8020
    0.5750
ri =
     0
     0
rmod =
    0.8020
    0.5750
ifail =
         0
```
